# **Case Study: Product Mix Optimization Problem**

**Background**

The ABC Company specializes in custom-building and selling two types of computing workstations for AI and deep-learning projects: **DL1** and **DL2**. These workstations cater to individual researchers, for-profit businesses, and universities over the internet. DL1 is the more affordable, less capable, and less sophisticated workstation, while DL2 is the premium, high-performance model.

Both products require significant time for hardware and software installation, configuration, and quality control before shipping to customers. The company's goal is to determine the **optimal product mix** (number of DL1 and DL2 units to produce) to **maximize its total monthly profit**.

**Problem Details**

1. **Workstation Features:**
   * **DL1** sells for $1,400 per unit with a **profit margin of 25%**.
   * **DL2** sells for $2,375 per unit with a **profit margin of 20%**.
2. **Labor Resources:**
   * The production of both DL1 and DL2 involves three tasks:
     + **Hardware installation**
     + **Software configuration**
     + **Quality control**
   * The time required for each task is detailed in **Table 2.1** below.

|  |  |  |  |
| --- | --- | --- | --- |
| Task | DL1 (hours) | DL2 (hours) | Monthly Total Available Hours |
| Hardware | 5 | 8 | 1,200 |
| Software | 3 | 4 | 800 |
| Quality Control | 2 | 2 | 400 |

1. **Demand Constraints:**
   * To maintain relationships with loyal corporate customers, the company must produce at least:
     + **50 units of DL1**
     + **30 units of DL2**
2. **Objective:**
   * Maximize the company's **total monthly profit** while respecting the resource and demand constraints.

**Tasks for Sensitivity Analysis**

**Based on the case study: Product Mix Optimization Problem, students are required to perform the following sensitivity analysis tasks after solving the primal problem:**

**Part 1: Solve the Primal Problem**

1. **Use the given data to:**
   * **Define the decision variables, objective function, and constraints.**
   * **Formulate and solve the primal problem to find the optimal number of DL1 and DL2 workstations to produce, and the maximum total monthly profit.**

**Part 2: Perform Sensitivity Analysis using LpMaximize, LpProblem, LpVariable, PULP\_CBC\_CMD**

1. **Objective Function Sensitivity (Reduced Costs):**
   * **Determine the reduced costs for DL1 and DL2.**
   * **Explain what the reduced costs mean:**
     + **If the reduced cost is zero, interpret why the product is part of the optimal solution.**
     + **If the reduced cost is non-zero, explain how much the profit margin must improve for the product to become part of the optimal solution.**
2. **Constraints Sensitivity:**
   * **For each constraint (Hardware, Software, and Quality Control), identify:**
     + **Shadow Price:**
       - **How much would the profit increase if the available hours for each resource were increased by one unit?**
       - **Which constraints are binding (fully utilized), and which are non-binding (unused resource)?**
     + **Slack:**
       - **How much of each resource is unused for the non-binding constraints?**
   * **Relate the shadow prices to managerial insights:**
     + **Which resource would provide the highest marginal increase in profit if additional capacity were added?**
3. **Demand Constraints Sensitivity:**
   * **Analyze the impact of increasing or decreasing the minimum production requirements (50 for DL1, 30 for DL2).**
   * **If the minimum demand changes by 1 unit, how does it affect profit?**

**Constraints Sensitivity Analysis**

1. **Constraint\_1**:
   * **Final Value**: −1.4×10^−5: This is effectively zero due to numerical precision, meaning the constraint is binding (fully used).
   * **Shadow Price**: 41.666667: Increasing the available resource for this constraint by one unit (RHS) would increase the profit by $41.67.
   * **Slack**: −0.0: There is no unused resource, confirming it is binding.
2. **Constraint\_2** :
   * **Final Value**: −133.333342: Indicates this constraint is not binding (unused resource exists).
   * **Shadow Price**: −0.0: No additional value is gained from increasing the resource availability because it is not fully utilized.
   * **Slack**: 133.33333: This is the amount of unused resource for this constraint.
3. **Constraint\_3** :
   * **Final Value**: −6.0×10^−6: Effectively zero, indicating this constraint is binding.
   * **Shadow Price**: 70.833333: Increasing the available Studios resource by one unit would increase the profit by $70.83.
   * **Slack**: −0.0: No unused resource, confirming it is binding.